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Risk Management Using Conditional Value-at-Risk (CVaR)

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Value-at-Risk (VaR), a widely used performance measure, answers the question: what is the maximum loss with a specified confidence level? Although VaR is a very popular measure of risk, it has undesirable properties such as lack of sub-additivity, i.e., VaR of a portfolio with two instruments may be greater than the sum of individual VaRs of these two instruments. Also, VaR is difficult to optimize when calculated using scenarios. In this case, VaR is non-convex, non-smooth as a function of positions, and it has multiple local extrema.

An alternative measure of losses, with more attractive properties, is Conditional Value-at-Risk (CVaR), which is also called Mean Excess Loss, Mean Shortfall, or Tail VaR. CVaR is a more consistent measure of risk since it is sub-additive and convex. Moreover, as it was shown recently [3,4], it can be optimized using linear programming (LP) and nonsmooth optimization algorithms, which allow handling portfolios with very large numbers of instruments and scenarios. Numerical experiments indicate that the minimization of CVaR also leads to near optimal solutions in VaR terms because CVaR is always greater than or equal to VaR. Moreover, when the return-loss distribution is normal, these two measures are equivalent [3], i.e., they provide the same optimal portfolio.

CVaR can be used in conjunction with VaR and is applicable to the estimation of risks with non-symmetric return-loss distributions. Although CVaR has not become a standard in the finance industry, it is likely to play a major role as it currently does in the insurance industry. Similar to the Markowitz mean-variance approach, CVaR can be used in return-risk analyses. For instance, we can calculate a portfolio with a specified return and minimal CVaR. Alternatively, we can constrain CVaR and find a portfolio with maximal return, see [2]. Also, rather than constraining the variance, we can specify several CVaR constraints simultaneously with various confidence levels (thereby shaping the loss distribution), which provides a flexible and powerful risk management tool.

Several case studies showed that risk optimization with the CVaR performance function and constraints can be done for large portfolios and a large number of scenarios with relatively small computational resources. For instance, a problem with 1,000 instruments and 20,000 scenarios can be optimized on a 300 MHz PC in less than one minute using the CPLEX LP solver. A case study on the hedging of a portfolio of options using the CVaR minimization technique is included in [3]. This problem was first studied at Algorithmics, Inc. with the minimum expected regret approach. Also, the CVaR minimization approach was applied to credit risk management of a portfolio of bonds [1]. This portfolio was put together by several banks to test various credit risk modeling techniques. Earlier, the minimum expected regret optimization technique was applied to the same portfolio at Algorithmics, Inc.; we have used the same set of scenarios to test the minimum CVaR technique. A case study on optimization of a portfolio of stocks with

CVaR constraints is included in [2].

REFERENCES

1. Andersson, F., Mausser, H., Rosen, D., and S. Uryasev (2000): Credit Risk Optimization With Conditional Value-At-Risk Criterion. Mathematical Programming to apper. (Draft can be downloaded: www.ise.ufl.edu/uryasev/and_mp.pdf; relevant Report 99-9 of the Center for Applied Optimization, University of Florida, can be downloaded: www.ise.ufl.edu/uryasev/pubs.html#t)

2. Palmquist, J., Uryasev, S., and P. Krokhmal (1999): Portfolio Optimization with Conditional Value-At-Risk Objective and Constraints. Research Report 99-14, Center for Applied Optimization, University of Florida. (Submitted to The Journal of Risk, Can be downloaded: www.ise.ufl.edu/uryasev/pal.pdf)

3. Rockafellar R.T. and S. Uryasev (2000): Optimization of Conditional Value-at-Risk. The Journal of Risk, Vol. 2, # 3. (Can be downloaded: www.ise.ufl.edu/uryasev/cvar.pdf; relevant Report 99-4 of the Center for Applied Optimization, University of Florida, can be downloaded: www.ise.ufl.edu/uryasev/pubs.html#t)

4. Uryasev, S. Conditional Value-at-Risk: Optimization Algorithms and Applications. Financial Engineering News, No. 14, February, 2000 (can be downloaded: www.ise.ufl.edu/uryasev/pubs.html#t).